2022 Chantilly Math Competition

Chantilly Math Competition Club

May 28, 2022

I Algebra

1. Solve $3p - 1 = 4 - 2p$ for p. (Normal/MC/4pts)

   A. -1 B. 0 C. $\frac{1}{2}$ D. 1

2. Find the y-intercept of the equation $x = \frac{1}{2}y + 1$. (Normal/MC/4pts)

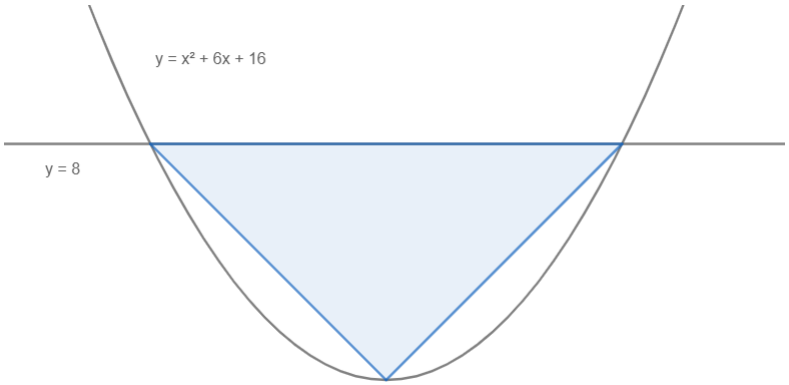
   A. -2 B. -1 C. $\frac{1}{2}$ D. 1

3. Let $f(x) = 2x + 4$. Find $f(f(f(-1)))$. (Normal/MC/4pts)

   A. 2 B. 8 C. 20 D. 32

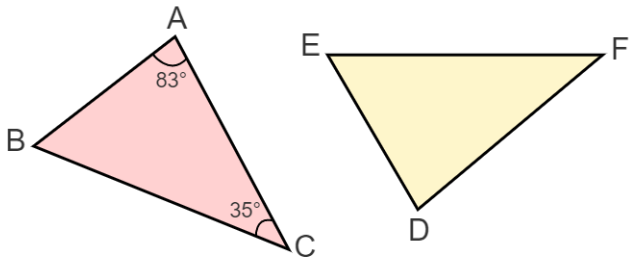
4. Find the sum of all coefficients and the constant of the expansion of $f(x) = (x + 1)^6$. (Hard/SA/6pts)

5. The parabola $y = x^2 + 6x + 16$ and the line $y = 8$ intersect at two points. Find the area of a triangle with points at those two intersection points and the vertex of the parabola. (Challenging/SA/8pts)



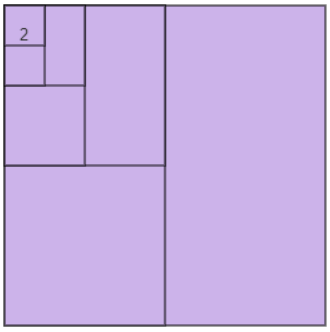
II Geometry

6. $\triangle ABC$ and $\triangle DEF$ are congruent triangles. Then, what is $m\angle E$?



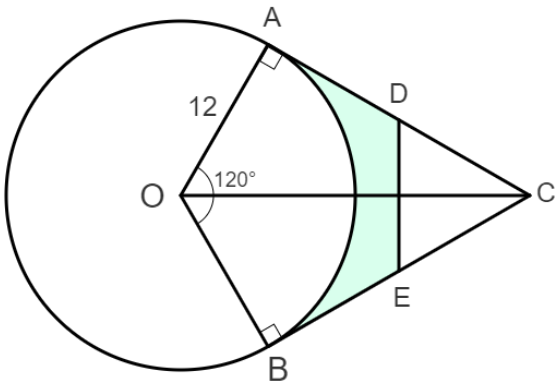
A. 35° B. 62° C. 72° D. 83°

7. Each of the vertical and horizontal lines inside of the square and rectangles divides their area in half, respectively. If the smallest horizontal line segment has a length of 2, find the area of the entire square. (Normal/MC/4pts)



A. 64 B. 128 C. 256 D. 512

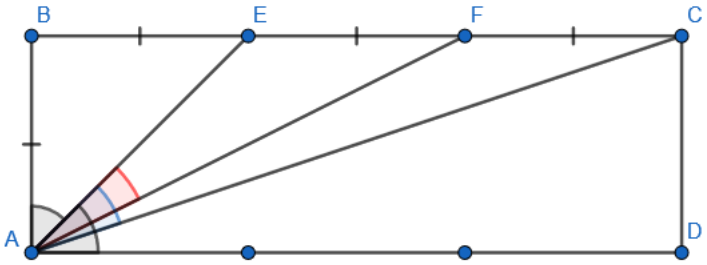
8. The circle with center O has a radius of 12 and is tangent to the line segments $\overline{\text{AC}}$ and $\overline{\text{BC}}$. If D and E are the midpoints of the line segments $\overline{\text{AC}}$ and $\overline{\text{BC}}$, respectively, and $m\angle AOB = 120°$, what is the area of the colored region? (Normal/MC/4pts)



A. $117\sqrt{3} - 48\pi$ B. $117\sqrt{3} - 8\pi$ C. $108\sqrt{3} - 48\pi$ D. $108\sqrt{3} - 8\pi$

9. How many times do the hour and minute hands meet between 0:00 and 24:00 inclusive? (Hard/SA/6pts)

10. Let $ABCD$ be a rectangle with $\overline{\text{BC}} = 3\overline{\text{AB}}$ and points E and F on $\overline{\text{BC}}$, such that $\overline{\text{BE}} = \overline{\text{EF}} = \overline{\text{FC}}$. Find $m\angle EAB + m\angle EAF + m\angle EAC + m\angle EAD$. (discard degree symbol) (Challenging/SA/8pts)



III Probability/Statistics

11. A problem is given to 3 students whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$, respectively. What is the probability that the problem will be solved by at least one student? (Normal/MC/4pts)

A. $\frac{3}{10}$ B. $\frac{4}{15}$ C. $\frac{11}{15}$ D. $\frac{31}{30}$

12. What is the probability to get at least three tails if you follow this rule: flip a coin, stop flipping if it is heads, flip again if it is tails. (Normal/MC/4pts)

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{3}{4}$ D. $\frac{7}{8}$

13. How many three-digit numbers N are there such that N is even but not a multiple of 3? (Normal/MC/4pts)

    A. 150 B. 300 C. 450 D. 900

14. There is an irregular six-sided die such that you have an $\frac{n}{21}$ probability to roll the number n (i.e. there is 5/21 chance to roll a 5). If the probability that sum of two dice rolls is 7 can be written as a fraction of the form $\frac{a}{21^2}$, find a. (Hard/SA/6pts)

15. The Fibonacci numbers are a sequence of numbers in which the next number is obtained by adding the two preceding numbers, where the first two are 0 and 1. (i.e. $0, 1, 1, 2, 3, 5, 8 \ldots$) How many multiples of 7 are there within first 1,000 terms of Fibonacci numbers inclusive? (Challenging/SA/8pts)

IV Number Theory

16. Find $((1 \circ 2) \circ 3)$ where $a \circ b = \frac{a+b}{a \cdot b}$. (Normal/MC/4pts)

    A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. 1 D. $\frac{3}{2}$

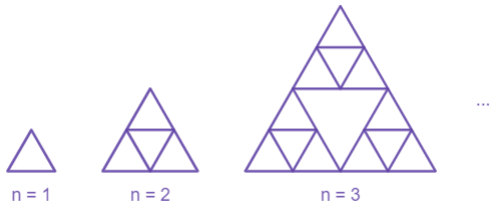
17. Let $f(x) = 6x^2 + 2x$. Find $f(1) + f(2) + f(3) + \ldots + f(10)$. (Normal/MC/4pts)

    A. 1800 B. 2080 C. 2420 D. 2860

18. If May 28, 2022, is a Saturday, what day of the week is May 28, 2048? (Normal/MC/4pts)

    A. Wednesday B. Thursday C. Friday D. Saturday

19. Find the total number of triangles of <u>any size</u> when n = 5. (Hard/SA/6pts)

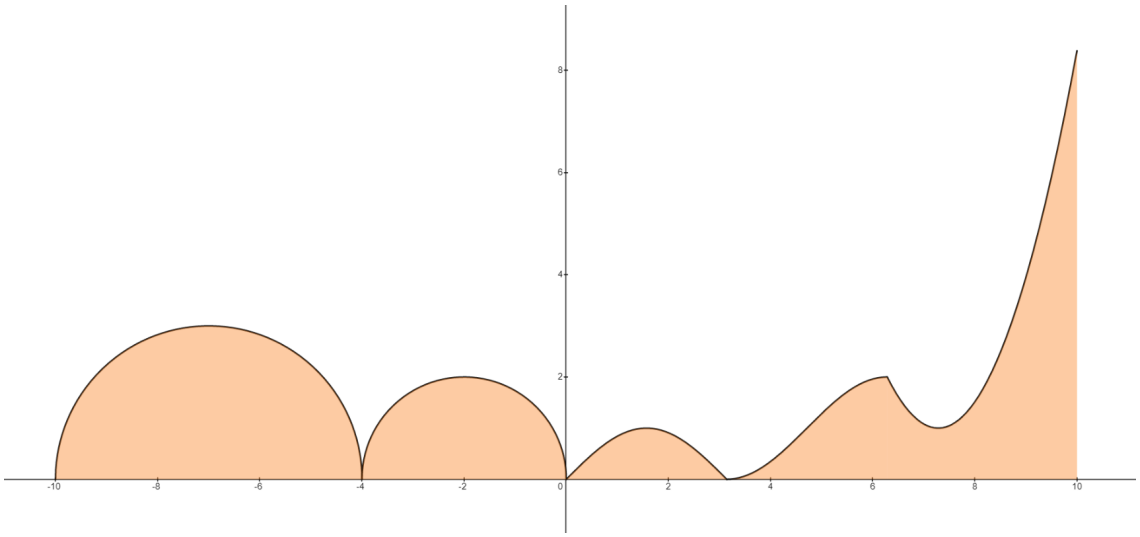


20. Solve $27^x - 18 \cdot 9^x + 81 \cdot 3^x = 0$ for real x. (Challenging/SA/8pts)

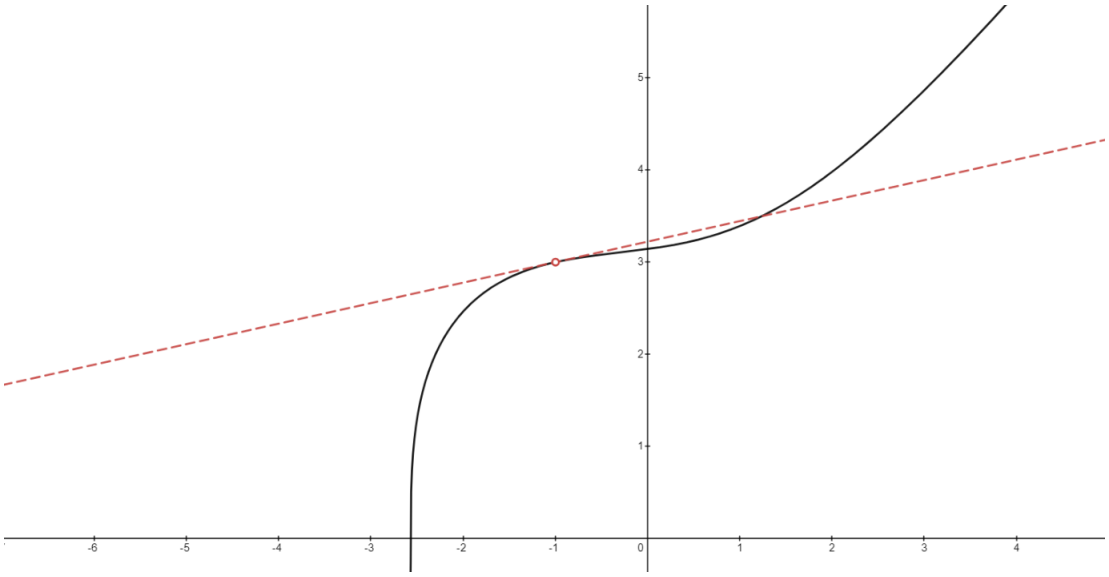
V Estimation

21. The graph of $f(x)$ is shown below. Estimate the area underneath the curve but above the x-axis from the interval $(-10, 10)$. (round to the ten thousandths place) (Estimate/SA/8pts)

$$f(x) = \begin{cases} (x+7)^2 + y^2 = 9 & y \geq 0 \\ (x+2)^2 + y^2 = 4 & y \geq 0 \\ \sin x & 0 \leq x \leq \pi \\ \cos x + 1 & \pi \leq x \leq 2\pi \\ (x - 2\pi - 1)^2 + 1 & 2\pi \leq x \leq 10 \end{cases} \tag{1}$$



22. The graph of $g(x) = (2x^3 + 2x^2 + 4x + 31)^{\frac{1}{3}}$ is shown below. Estimate the slope of the tangent line of $g(x)$ at $x = -1$. (round to the ten thousandths place) (Estimate/SA/8pts)



VI Advanced

23. Let $g(x)$ be the greatest power of 2 that divides x without a remainder. ($g(x)$ is a factor of x) For example, $g(20) = 4$ because $20 = 2^2 \cdot 5 = 4 \cdot 5$ and $g(32) = 32$ because $32 = 2^5 = 32 \cdot 1$. Then, find the general formula of $S(n) = g(1) + g(2) + \ldots + g(2^n)$ in terms of n. (Advanced/FR/10pts)

24. Let $11x + 7y = 5$. How many ordered pairs of integers x and y if $|x| + |y| < 30$? (Advanced/FR/10pts)

25. Let $I(n)$ be the size of one internal angle of polygon with n angles in radians. For example, $I(3) = \frac{\pi}{3}$ and $I(4) = \frac{\pi}{2}$ because they are the size of one internal angle for an equilateral triangle and a square, respectively. Then, find the general formula for $S(n) = I(3) + I(4) + \ldots + I(n)$ in terms of n. (Advanced/FR/10pts)