2025 Chantilly Math Competition

Middle School Division

Competition Instructions

1. Do not open the test until instructed.
2. No computational aids other than pencil/pen are permitted.
3. Write answers in designated boxes on answer sheet.
4. All answers must be nonnegative integers: $\{0, 1, 2, \ldots\}$.
5. Points per question range from 5 to 12.
6. The test consists of 25 short-answer questions to be solved in 100 minutes.
7. The final question will be used for tie-breaking.
8. There is no penalty for incorrect answers.
9. You will be graded on your answers—not your work.
10. If you have any questions, ask your proctor.
11. **Notation/Definitions:**
    - The dot $\cdot$ denotes multiplication: $a \cdot b = a \times b$.
    - $n!$ denotes the factorial of n: $n! = n \times (n-1) \times \cdots \times 1$.
    - Two integers a and b are *relatively prime* if and only if $\gcd(a, b) = 1$.

1. (5 pts) Saul's age is currently a quarter of Paul's age. In 10 years, Saul's age will be half of Paul's age. How old is Paul?

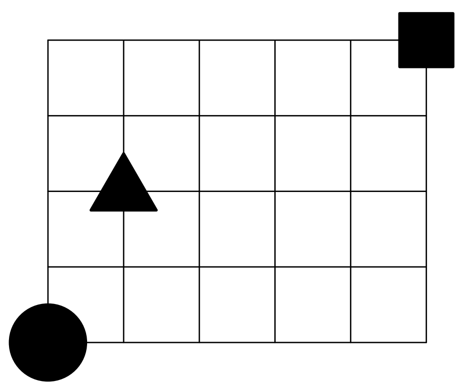
2. (5 pts) Ella loves bubble tea, but there are so many toppings to choose from! One serving of pearls costs \$1, pudding costs \$2, and jelly costs \$3. If Ella decides to spend exactly \$6 on toppings, how many ways can she customize her bubble tea? She can get more than one serving of any topping.

3. (5 pts) A parking lot has dimensions of $20' \times 30'$. How many cars of $4' \times 3'$ can fit inside the lot if every car's long side is parallel to the parking lot's long side?

4. (5 pts) Five distinct positive integers are written in a line in increasing order. If the average of any 2 adjacent numbers in the line is a prime integer, compute the minimum possible sum of the five integers in the line.

5. (6 pts) Given that real numbers x, y, and z satisfy $1^x = y$, $2^y = z$, and $3^z = x$, find $x^y + y^z + z^x$.

6. (6 pts) The diagram below shows a 5×4 rectangular grid. Consider paths from the starting point $(0, 0)$ (marked by the circle) to the destination $(5, 4)$ (marked by the square). Movement along the grid lines is restricted to 1 unit rightwards or 1 unit upwards at each step. How many such paths avoid passing through the point $(1, 2)$ (marked by the triangle)?



7. (6 pts) There exist points A, B, C, D, and E in the plane such that $AB = 75$, $BC = 72$, $CD = 16$, $DE = 12$, and $\angle ACB = \angle ACE = \angle CDE = 90°$. Find the length of segment AE.

8. (6 pts) If $a, b,$ and c are integers such that $a + b = -11$ and $bc = 20$, compute the maximum possible value of $\max(ab, bc, ca)$.

9. (7 pts) Compute the sum of $|P(3)|$ over all monic cubics $P(x) = x^3 + ax^2 + bx + c$ such that

$$P(a) = P(b) = P(c) = 0.$$

10. (7 pts) John writes all distinct permutations of the string `JJOOHHNN` on his whiteboard. How many permutations on his whiteboard contain the substring `JOHN`?

11. (7 pts) Let $MONKEY$ be a hexagon and CAT be a triangle inside $MONKEY$ such that $MOAT$, $CANK$, and $YECT$ are all squares. If CAT is an equilateral triangle with side length 2, then the area of $MONKEY$ can be expressed as $\sqrt{a} + \sqrt{b}$ for positive integers a and b. Compute $a + b$.

12. (7 pts) Compute the number of ordered triples of positive integers (x, y, z) that satisfy the equation $x^1y^2z^3 = 2025$.

13. (8 pts) Let S be the sum of the real solutions to

$$x^4 + 6x^3 - 3x^2 + 6x + 1 = 0$$

If $S = a + b\sqrt{c}$ for integers a, b, and c where c is positive and square-free, find $|a| + |b| + |c|$.

14. (8 pts) How many distinct ways are there to distribute 15 identical candies to 4 distinct children—Alice, Bob, Charlie, and David—subject to the following conditions:

   i. Alice must receive at least 1 candy.
   ii. Bob must receive at most 3 candies.
   iii. Charlie must receive an even number of candies (zero is considered an even number).
   iv. David must receive an odd number of candies.

15. (8 pts) Points A, B, C, and D all lie on a circle. Chords AC and BD intersect at point E. Given that $BC = 6$, $AE = 4\sqrt{3}$, and $\angle BAC = \angle BCA = 30^\circ$, find the length of AD.

16. (8 pts) Let S be the set of all tuples of positive integers (a, b, c), such that $abc = 2^3 \cdot 3^3 \cdot 5^3$. Find the remainder of

$$\sum_{(a,b,c)\in S} (a + b + c)$$

when divided by 1000.

17. (9 pts) Real numbers a, b, c, where $a < b < c$, satisfy the following system

$$a^2bc + ab^2c + abc^2 = 90$$

$$3abc + a^2b + a^2c + ab^2 + b^2c + ac^2 + bc^2 = \frac{255}{2}$$

$$a^2b^2c + ab^2c^2 + a^2bc^2 = 204$$

Find $2a + b + c$.

18. (9 pts) A standard deck of 52 playing cards contains 4 Kings and 4 Queens. The expected number of cards between the topmost King and topmost Queen can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $m + n$.

19. (9 pts) Let A, B, C, D, E, F, and O be points in the plane. Line AB is tangent to a circle with center O and B is on the circle. Line AD passes through point O and intersects the circle at points C and D. Line AF intersects the circle at points E and F. Given that $AE = 5$, $EF = 2.2$, $AD > AC$, $AF > AE$, and the radius of the circle is 2.5, find $AB + AC$.

20. (9 pts) Let b be the smallest positive multiple of 2025 whose digits are all 0 or 1. Compute the decimal value of b interpreted as a binary number.

    For example, if $b = 10110$, then your answer should be $2^4 + 2^2 + 2^1 = 22$.

21. (12 pts) Let the roots of $x^2 + ax + b = 0$ be r and s where $r \neq s$. If $|r - s| = b$ and $a + b = r + s$, then $|ab|$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $m + n$.

22. (12 pts) Let $\triangle ABC$ be a triangle with $AB = 13$, $BC = 14$, and $CA = 15$. Let (ABC) be the circumcircle of $\triangle ABC$. Let T be on BC such that $AT \perp BC$. Let I be the midpoint of BC. Let $Y \neq A$ be an intersection of AT and (ABC). Let $L \neq A$ be an intersection of AI and (ABC). If the area of quadrilateral $TILY$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n, find $m + n$.

23. (12 pts) Let $r(n)$ be the number formed when the digits of n are reversed. Compute the sum of all prime numbers p such that $p < 500$ and $p + r(p)$ is a perfect square.

24. (12 pts) Let $f(x)$ denote the number of permutations of the set $\{1, 2, \ldots, x\}$ such that for every position n where $1 \leq n \leq x$, the element at position n is either 1 or divisible by n. Compute $|f(20) - f(25)|$.

25. (12 pts) Compute the number of functions $f : \{1, 2, 3, 4, 5, 6\} \to \{1, 2, \ldots, 2024\}$ such that

    - $f(i) \mid f(j)$ whenever $i \mid j$, and
    - $f(x) \mid 2024$ for all $x \in \{1, 2, 3, 4, 5, 6\}$,

    where $a \mid b$ if and only if a is a divisor of b.

TB. (Tiebreak) Nischal has 2025 squares in a row. He wants to color each of them red, green, or blue. Estimate the sum of the digits of the number of colorings where no 3 consecutive squares are the same color.

2025 Chantilly Math Competition - Answer Sheet
Middle School Division

Name: ______________________________ **Grade:** __________

Email: ______________________________

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

TB